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Developing Abstracting and Generalizing Thinking for Middle School Students in Teaching the Topic of Equations and Non-Equations

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ABSTRACT: In the Vietnam general education curriculum, Mathematics subject has an important position, being a tool for many other subjects. Mathematics has great potential to help students develop their intellectual abilities and qualities, training them to think abstractly and generalize. Furthermore, Mathematics has its roots in practice and is the "key" in most human activities, as well as in learning other subjects. In teaching mathematics, teachers not only guide students to acquire knowledge and skills but also train them to think and learn. Therefore, training in abstract thinking and generalization thinking is necessary to help students not only understand and remember math more deeply but also develop problem-solving, logical thinking and judgment skills, thereby helping them develop more comprehensively in the learning process and life.

KEYWORDS- abstracting thinking, generalizing thinking, Middle School, Equations, Non-equations.

I. INTRODUCTION

Generalization and abstraction are two important operations in human thinking in general, and mathematical thinking in particular. These operations are closely related to each other. Abstraction enables broader generalization, leading to a deeper understanding of objects and phenomena. In teaching Mathematics, concepts are often formed based on generalization and abstraction processes. Furthermore, knowledge systems are established through the connection of generalization and abstraction processes. Many authors worldwide have studied the operations of thinking and their relationships. According to G. Polya, thinking operations include analysis, synthesis, comparison, analogy, generalization, and specialization. He suggests that generalization involves transitioning from studying a given set of objects to studying a larger set that includes the original 14). M.N. Sacdacop analyzed students' thinking processes from intuitive cognition to logical cognition. He emphasizes skills and techniques developed in thinking such as synthesis, comparison, abstraction, generalization, and specification 16). Edward de Bono considers thinking as one of the fundamental and essential skills of humans. He has proposed methods like Lateral thinking and the Six Thinking Hats method 2).

N. Hashemi argues that generalization can be defined from a mathematical perspective as "seeking a larger image," or considering limited groups moving to larger groups, or extending the concept to a larger scope to explore it 5). According to M. Mitchelmore, the essence of abstraction in mathematics is its self-contained nature, separate from the physical and social world. An abstract mathematical object only has meaning within the system in which it is defined 12). G. Harel states that mathematicians and mathematics educators use the terms generalization and abstraction with different nuances of meaning. Each term represents both a process and the result of that process 6). In Vietnam, there is also research work on this issue. Nguyen Ba Kim refers to these not as cognitive operations but as fundamental intellectual activities including analysis, synthesis, comparison, analogizing, abstracting, generalizing, and specifying 8).

The process of abstraction involves moving from a set of objects to a larger set containing the original set by highlighting common characteristics of the elements in the original set. Abstraction is about separating essential characteristics from non-essential ones. The distinction Hoang Chung, the mental operation includes analysis, synthesis, comparison, generalization, abstraction, and specification. Generalization is seen as using the mind to extract what is common among objects phenomena, and events. To generalize, it is often necessary to compare multiple objects, phenomena, or events with each other. During this process, we discard their different attributes, the attributes that distinguish those objects or phenomena from each other, and focus only on the elements that are separated; this is the process of abstraction 4).

Nguyễn Phu Loc defines abstraction as the process of using the mind to eliminate aspects, attributes, secondary relationships, and unnecessary connections, and retaining only the essential elements for thinking 11).

In teaching mathematics, abstract thinking and generalization are fundamental and essential skills. Abstract thinking helps students recognize and understand mathematical concepts at a more abstract level, not just limited to specific examples. This enables

them to identify patterns and relationships between concepts, allowing them to apply them to different situations. Generalization helps students recognize and apply general rules or problem-solving models to various situations, saving time and energy in problem-solving. Additionally, abstract thinking and generalization help students develop problem-solving analysis skills, identify common patterns, and apply problem-solving methods to new situations. This not only deepens their understanding and long-term memory of mathematics but also promotes holistic development in learning and life. Therefore, besides teaching the subject matter comprehensively, training students in critical thinking is significantly important in enhancing the quality of teaching and learning various content in Mathematics at middle school.

II.CONTENTS

2.1. Some general issues about abstract thinking and generalization

2.1.1. Thingking

The objective world is very rich and diverse. To master life, people need to deeply understand the unknown, delineate the essence, relationships, and laws of their impact. This process is called thinking. There are many ways to understand the concept of thinking, for example:

According to psychology, thinking is a special attribute of highly organized matter - the human brain: "Thinking reflects the internal properties, essence, the law-based relationships of phenomena that we did not previously know" 7). Thus, this process of reflection is indirect and abstract, arising from practical human activities, originating from sensory perception but transcending its limits.

According to V. I. Lenin, thinking is a deeper, more loyal, more complete reflection of the natural world, delving infinitely closer to objective truth. "People's thinking - delving infinitely, from imagination to essence, from first-level essence, if possible, to second-level essence... to infinity" 10).

According to Ast Costa, the author of one of many books on thinking, believes that "Thinking is our perception when we receive data, information that occurs in relationships." In short, "We think"18).

Thinking is the ability to think, understand, and solve problems based on information and knowledge that a person has learned and experienced. It is not only limited to performing calculations or logical reasoning but also includes the ability to be creative, analyze, evaluate, and apply knowledge to reality. In summary, thinking can be understood as a psychological process that reflects the inherent attributes, relationships, and internal relationships with the laws of objects, phenomena in objective reality that we did not know before.

2.1.2. Characteristics of thinking

2.1.2.1. Problematic nature of thinking

In any situation, the process of thinking does not just happen. To stimulate thinking, two conditions must be met simultaneously: First, thinking only emerges when encountering situations with "issues." These are scenarios containing a new purpose, a new problem that requires a new solution method where existing knowledge and past actions are necessary but insufficient. To solve the problem, achieve a new purpose, individuals need new methods, new approaches. In other words, individuals need to think.

Second, situations with issues stimulate human thinking. Situations with problems must be fully recognized by individuals and transformed into their own tasks. Understanding the conflicts within the problem that the subject needs to resolve and possessing the necessary knowledge to solve it is the basis for thinking to arise. In other words, the situation or scenario not only triggers the need for awareness but also must instill belief in the individual's problem-solving abilities.

2.1.2.2. Indirectness of thinking

Human beings reflect objects and phenomena through their senses based on intuitive cognition. Humans do not perceive the real world directly but rather indirectly. The indirect nature of thinking is specifically demonstrated as follows:

- The indirect nature of thinking is manifested in language. Humans use language to think.
- During the thinking process, humans utilize tools, means (such as thermometers, machines, etc.) and employ the results of cognition (rules, formulas, laws, concepts, etc.) of humankind and personal experience in the thinking process (analysis, synthesis, comparison, generalization, etc.) to perceive the inner essence of objects, phenomena, and the laws between them.

Therefore, thanks to this indirect nature, human thinking is expanded limitlessly in terms of cognitive ability. Thinking not only reflects what is happening in the present but also reflects both the past and the future.

2.1.2.3. Abstract and general nature of thinking

Thinking does not reflect specific and individual phenomena. Thinking delves into objects and phenomena to outline common properties, relationships, law-like connections, reflecting the most common and essential characteristics of a type of object. In other words, thinking reflects objective reality. To generalize objective reality, thinking must abstract from objects, phenomena, individual attributes, specific signs, retaining only the common essential attributes for many objects and phenomena, then based on that, generalize individual objects, phenomena with common essence into a group, a type, a category. Therefore, thinking reflects the

common essence of many objects and phenomena into a group, a type, a category, while abstracting from objects, phenomena, specific individuals. Hence, thinking is both abstract and general. This is also a characteristic that we particularly focus on.

2.1.2.4. Thinking is closely related to language

Thinking is problematic, indirect, abstract, and general because it is closely related to language. Without language, the thinking process itself does not occur simultaneously with the products of thinking (concepts, judgments, etc.) not being subject to and accepted by others. Language fixes the results of thinking, thus making it possible to objectify them for others and the thinking subject itself. Conversely, without thinking, language is just a meaningless string of sounds. Thinking uses language as a tool because:

- In the mind, the subject can only become aware, perceive a problematic situation thanks to the language process.
- In the course of thinking, humans use language to perform operations: analysis, synthesis, comparison, generalization, abstraction - language participates in the thinking process.
- The results of the thinking process are expressed by language.

In summary, thinking and language have a close relationship with each other. However, language is not thinking; language is merely a tool of thinking.

2.1.2.5. Thinking is closely related to sensory perception

Thinking and emotional perception belong to two different cognitive levels, yet they are closely related, complementing and influencing each other in unified, dialectical activities as follows:

- Thinking often originates from emotional perception, triggering the emergence of problem situations that stimulate thinking. According to psychologist X. L. Rubinstein: "The content of emotions is always present in abstract thinking, acting as a foundation for thinking" 15).
- Conversely, thinking and its outcomes strongly impact the reflective ability of emotional perception, making human sensory abilities more sophisticated and acute; shaping selective and meaningful perception. Ph. Angghen wrote: "Through our eyes, not only do we have different sensations but also our thinking activities" 1). Additionally, thinking adjusts and corrects the errors of emotional perception.

In the study of Mathematics, this characteristic is exemplified: To understand the content or prove a problem, the first step is to rely on emotional perception to identify requirements or hypotheses in this direction or another, leading to observations, followed by verification through thinking activities to reach conclusions.

2.1.3. Operations of thinking

The thinking process operates as an activity. Essentially, thinking is a personal process of performing specific intellectual operations to solve problems. These operations are internal rules of thinking, including analysis - synthesis, comparison, abstraction - generalization.

2.1.3.1. Analysis and synthesis thinking

According to 12), analysis is the process of using the mind to divide the object of perception into parts, different components to reveal the attributes, and characteristics of the object of perception, or to identify the parts of a whole by comparing, classifying, contrasting, making the whole explicit. Synthesis is the process of using the mind to integrate, arrange, or combine the parts, components, attributes of the object of perception that have been separated through analysis into a coherent whole in order to perceive the object in a more comprehensive, holistic manner. In thinking, synthesis is considered a creative operation 12). Analysis and synthesis are two fundamental activities of thinking. Other activities take place based on analysis - synthesis. They have an intimate and inseparable relationship: Analysis is carried out in the direction of synthesis based on the results of analysis.

2.1.3.2. Compare thinking

According to 17), "Comparison is the process of using the mind to determine similarities or differences, identities or non-identities, equalities or inequalities between perceptual objects (objects, phenomena)." The act of comparison is closely related to the process of analysis and synthesis. Comparison helps us identify the essential similarities or differences of objects.

2.1.3.3. Abstraction – generalization thinking

Abstraction: It is the process of using the mind to eliminate aspects, attributes, secondary relationships, and only retain the characteristic elements, the essence of the perceived object 3).

Generalization: It is the process of using the mind to integrate multiple different objects into a group, a type based on attributes, relationships, common connections, the essence of things, phenomena. The result of generalization is to produce a common characteristic of a series of similar objects or create new perceptions in the form of concepts, laws, rules 7). Abstraction and generalization are closely related, complementing each other like analysis and synthesis but at a higher level. Therefore, to develop abstract and generalization abilities, it is necessary to first practice the ability to analyze and synthesize.

2.1.4. The role of thinking training in teaching Mathematics in middle school

Developing Abstracting and Generalizing thinking for Middle School Students in Teaching the Topic of Equations and Non-equations

Mathematics is considered as the "intellectual sport" because among the subjects in mainstream education, including middle school level, Math is the most potential subject for developing students' thinking skills. It serves as both a fundamental subject and a highly applicable one.

Mathematics plays a crucial role in achieving the general goals of basic education. It contributes to personal development, providing opportunities for students to construct knowledge and practice essential mathematical skills. It also enhances the development of general intellectual abilities such as analysis, synthesis, comparison, generalization, abstraction, fostering qualities like attentiveness, accuracy, discipline, critical thinking, and creativity.

Thinking plays a significant role in human life and cognitive activities such as:

- Expanding the limits of perception helps individuals grasp a wide range of knowledge and understand relationships across various fields. Thinking not only solves immediate tasks but also addresses future challenges by understanding the laws and dynamics of nature and society. In teaching Mathematics, critical thinking serves the purpose of honing cognitive skills: analysis - synthesis, comparison, abstraction - generalization. This thesis focuses on cultivating abstract thinking - generalization in teaching the topic of PT. Mathematical thinking can be divided into two levels: Level 1 (Reconstructive): This level refers to mathematical learning ability and consists of 3 stages: + Ability to absorb knowledge + Infer and apply acquired knowledge + Demonstrate relationships. Level 2 (Creative): Refers to the ability for creative mathematical activities and discovering new results, problem-solving methods beyond conventional patterns.

2.2. Training abstract thinking and generalization for middle school students in teaching mathematics

2.2.1. Orientation for proposed measures

Building measures to train students in abstract thinking and generalization in teaching the topic of Equations for middle school students needs to adhere to specific orientations:

- Alignment with the nature of Mathematics: Mathematics is a highly abstract science, and abstract mathematical thinking is always associated with generalization. In teaching Mathematics at the middle school level, teachers need to focus on training students in operations, problem-solving skills through the application of calculations and methods to solve various math problems; Developing skills in problem analysis, reasoning, proposing appropriate solutions, generalizing solution steps... thereby training students in abstract thinking and generalization.

- Respecting the theoretical foundation of the research issue: Measures to train students in abstract thinking and generalization for middle school students are built on the theoretical framework of abstract and general thinking. Each measure needs to identify the potential for developing students' thinking.

- Ensuring the feasibility and practicality of the measures: The measures must be in line with the objectives and content of the Math curriculum for 8th and 9th grades; suitable for the conditions, circumstances, and the direction of educational innovation in Vietnam in general and mathematics education in particular.

2.2.2. Some pedagogical measures to train abstract thinking and generalization for middle school students in teaching Equations

2.2.2.1. Developing abstract thinking and generalization in teaching and forming new concepts

a) Basis of the measure

According to the author Nguyen Ba Kim (2002), the concept is a form of thinking that clarifies the characteristics of a certain class of objects. A concept can be considered from two aspects: "external" and "internal." The external aspect is the class of objects that defines the concept, while the internal aspect is all the common attributes of this class of objects.

In another book by authors Nguyen Ba Kim and Vu Duong Thuy (1992), it is mentioned that "When we define a concept, its internal and external aspects are already determined. The external aspect of the concept will be further clarified through concept division. Understanding concept division is an indication of mastering mathematical concepts as well as concepts from any other subject" 9).

Therefore, students need to distinguish between the two concepts of "external" and "internal" in order to argue in calculus, geometry, and solve various other problems based on case differentiation (Nguyen Ba Kim, 2002).

Thus, analyzing the internal aspect helps students understand the essence and attributes of the concept, while concept division helps students understand more specifically the external aspect of the concept. This, in turn, enables students to apply the concept in solving mathematical problems. This approach aims to help students develop analytical skills, synthesize concepts to deeply understand the essence of the issue.

To help students excel in Mathematics, it is essential not only to require them to master and apply various problems and basic forms but also to teach them how to develop these into new types of problems with higher reasoning levels. This approach aims to enhance students' critical thinking skills, enabling them to practice abstract and generalized thinking processes effectively.

Therefore, it is important to train students in comparing objects and phenomena from multiple perspectives, allowing them to grasp the essence of the issue and apply it to similar or more complex math problems. This method helps students understand different aspects, facilitating reliable results in generalization and prediction processes.

This strategy focuses on honing students' skills in analysis, synthesis, reasoning, and comparison, transitioning from learned concepts to solving new types of problems. It assists students in solving various math problems and applying knowledge to tackle more intricate scenarios.

Moreover, the purpose of this approach is to cultivate skills such as analogical thinking, specialization, abstraction, generalization, and other essential skills for students. This process empowers students to innovate and create their math problems, leading to a diverse and rich exercise system.

b) How to carry out measures

Training methods for abstract thinking and generalization in education to form new concepts consist of 3 steps as follows:

Step 1: Identify the signs, attributes of the concept, distinguishing between essential and characteristic attributes. Provide examples that satisfy the concept definition and counterexamples. Divide the concept into parts based on each sign, attribute.

Step 2: Study the properties of the separated parts. Determine the relationship between that concept and other concepts previously learned.

Step 3: Combine the relationships into a complete concept along with its properties.

This teaching approach can be implemented for students through the following activities:

Activity 1: Create situations for students to engage their cognitive abilities with concepts.

Develop problem scenarios that stimulate students' cognitive needs regarding the concept of equations. Organize activities for students to solve the presented problems to discover the essential attributes of the concept.

Before explicitly teaching equations, students have been subtly introduced to various equation types, including solving them. Therefore, teachers can use mathematical problems, problems related to identities, functions, algebraic expressions, values of algebraic expressions, etc., to create scenarios that help students approach the concept of equations.

Activity 2: Constructing the Definition of Concepts

- The teacher presents a new situation, organizes students to carry out activities such as analysis, comparison, and reference... selecting objects with intrinsic characteristics of the concept.
- Through generalization, students present the definition of the concept.

Activity 3: Practice, Reinforcement, and Application of Concepts

- The teacher organizes activities for students to identify concepts in practical situations in mathematics and daily life.
- Students construct examples that demonstrate newly formed concepts.
- Students apply the concepts learned in specific situations such as solving math problems, proving theorems, constructing other concepts, and applying them in practice.
- Organize students to consider specific and general cases.
- Arrange the logic of concepts and the relationship between new concepts and previously learned concepts.

2.2.2.2. Practice abstract thinking - generalization in determining the solution steps for an equation

a) Basis of the measure

Currently, the entire education sector in general, and specifically secondary education, is implementing the requirement to innovate teaching methods by enhancing students' learning enthusiasm, making classroom teaching sessions "smooth, natural, and effective." To meet this requirement, teachers need suitable teaching methods and forms that align with the psychological characteristics of each age group and students' cognitive levels, enhancing teaching effectiveness to meet the overall educational innovation demands of the country and specifically at the grassroots level.

In the mathematics curriculum, particularly in the topic of linear equations at the secondary level, plays a significant role. Through solving problems, students can see many mathematical concepts intertwined with real-life situations, in human activities, understanding the dialectical relationship between events, between what is given and what needs to be found. Solving problems helps students develop critical thinking skills and new human virtues: resilience, carefulness, working with a plan, having a basis for judgment, self-assessment of their work results, independence in thinking, creativity, enabling students to apply knowledge, practice calculation skills, and communication skills. Additionally, through students' problem-solving, teachers can easily identify their strengths and weaknesses in knowledge and skills to help students enhance their achievements and address existing shortcomings.

Solving a math problem requires a specific process, which also helps develop students' critical thinking skills. Prior to teaching, it is essential to discuss with colleagues and teachers to align on methods, establish processes, and share experiences in teaching that type of math problem. Additionally, investing time in thoroughly researching exercises for each type of problem, from lesson content to practice exercises, from textbook problems to workbook exercises, is crucial for developing a suitable, concise teaching method that is easy for students to grasp. Teachers should speak less, carefully select additional exercises to enhance knowledge for students of varying abilities, and anticipate common challenges students may face while solving problems. All teacher preparations should be clearly outlined in lesson plans, covering all steps, requirements, and demonstrating the roles of both teachers and students during math problem-solving sessions.

b) How to carry out measures

- Understand the content of the problem.

- Find a solution: Explore and discover a solution through speculative thinking. Transform the given information, modify what needs to be found, relate the given or required information to existing knowledge, connect the problem at hand with a similar one. Then, verify the solution by carefully reviewing each step taken and cross-referencing with relevant knowledge. Explore alternative solution methods, compare them to choose the most reasonable one.

- Present the solution: Organize the discovered solution method into a sequence of steps and execute them accordingly.

- Verify and delve deeper into the solution: Check the results, review all solution steps, derive methodological knowledge to solve a specific type of problem. Research the applicability of the solution results, study how to solve similar types of problems, expand or reverse the problem.

With this approach, students will specifically follow these steps:

- Students follow the process under the guidance of the teacher by engaging in situations where the teacher directs them to perform each step of the operational process through explanations, presentations, and questioning to help students understand the execution of each step.

- Students independently carry out the steps of the process with guidance from the teacher when necessary.

- Students proficiently execute the steps of the process independently.

Specifically, the teacher guides students to grasp the solution steps and categorize different types of problems under each specific category.

2.2.2.3. Building a system of exercises on Equation Topics to support the training of abstract thinking - generalization for middle school students

a) Basis of the measure

A set of exercises is understood as a collection of exercises linked together by defined relationships, usually related to teaching objectives, mathematical content, or the level of mathematical thinking development. Constructing an exercise system aims to support cultivating abstract thinking - generalization skills for middle school students.

b) How to carry out measures

Building an exercise system can be structured as follows:

- Construct and organize problems according to mathematical forms: Teachers retain the essential attributes, distinctive features of the mathematical form to create new similar exercises by modifying some conditions or elements in the problem (changing numbers, rearranging equations, etc.).

- Developing an exercise system that can be transformed into familiar forms through equivalent transformations: These are exercises that may not initially resemble the general form of the learned mathematical concept but through a few steps of transformation, students can arrive at a familiar form. This not only helps students practice abstract thinking and generalization but also contributes to fostering creative and flexible problem-solving skills.

- Establishing practical exercise systems to help students apply mathematical knowledge on the topic of mathematics to solve real-world problems: This is one of the key objectives of mathematics and has been emphasized more in the curriculum post-2020. This exercise system requires students to be able to model real-life situations (through abstract reasoning) into mathematical problems while also enabling them to apply the knowledge and skills learned in real-life situations.

III. CONCLUSIONS

Generalization and abstraction can be considered as two essential operations in thinking in general, especially in mathematical thinking of students. They are closely related and complement each other in the process of intellectual emergence and development of learners. Generalization can be seen as the beginning stage and abstraction as the concluding stage in the journey towards the truth of a mathematical concept. Through abstraction, generalization can be broader, allowing students to perceive the essence of objects and phenomena more profoundly. Training and utilizing abstract and generalization activities are necessary for middle school students. Engaging in thinking processes in this direction contributes to the development of students' thinking abilities.

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