

An Innovative Model to Promote Secondary Students' Critical Thinking Skills in Algebra Learning



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ABSTRACT: This research aims to explore a learning model based on 21st-century skills and the Australian curriculum to cultivate students' critical thinking skills in junior secondary mathematics algebra learning. This study used a mixed method. Quantitative data was collected through a questionnaire that provided information about perceptions of the challenges in learning algebra. Qualitative data was collected through semi-interviews with secondary mathematics teachers, which provided an opportunity for teachers to evaluate their understanding of critical thinking skills for algebra, it further clarified teachers' expectations of mastering the key algebraic contents. Based on the data from the questionnaire and semi-interview, a practical model for embedding critical thinking skills and strategies in learning algebra was designed. The mathematical curriculum in Victoria and Australia were explored as they provided the basis of the model.

KEYWORDS: Critical thinking Algebraic learning Critical- Algebraic model Algebraic learning framework

I. INTRODUCTION

Critical thinking has been regarded as an important and necessary educational objective. The importance of critical thinking can be in many educational documents and international assessments such as the National Council for Excellence in Critical Thinking ([1]) and the Program for International Students Assessment (PISA) ([2]). NCECT states that critical thinking is defined as an intellectually disciplined process, which involves conceptualizing, applying, analysing, synthesizing, and evaluating information.

Critical thinking is a skill that makes an individual think, question, and challenge ideas, generate solutions to problems and make intelligent decisions based on analysis and evidence ([3], [4]). In mathematics, critical thinking may involve logical reasoning and skills in separating facts from opinions ([5]). When solving problems, critical thinkers may have the ability to understand logical connections between ideas, detect errors in inductive and deductive reasoning problems, and make valuable decisions ([5]). All these skills are included in critical thinking. Critical thinking promotes students' thinking level and benefits them in learning mathematics content. Thus, promoting and developing students' critical thinking skills should be addressed in mathematics education ([5]).

II. LITERATURE REVIEW

1. Teaching critical thinking in mathematics

Critical thinking is an essential skill applied across the curriculum and beyond the classroom ([6]), it helps improve students' performance in any subject, especially in mathematics. Mathematics learning is a cognitive developmental process like the developmental stages in Jean Piaget's Cognitive Developmental Theory ([7],[8]). According to Piaget's theory, cognitive structure helps children build meaning from the environment; the evolution between each stage depends on mastering the previous stage ([8]). Mathematics is a developmental process, and mastering each stage can predict the success of students in the next stage. Students construct new ideas based on their prior knowledge and begin to think at a higher cognitive level. Critical thinking enables students to not only think critically but also use cognitive models to construct new meaning in their learning ([9]). There are many studies about critical thinking in mathematics, such as discussing the importance of critical thinking in mathematics ([10],[11],[12]), increasing/developing critical thinking skills in mathematics class ([13],[14]). In a study investigating critical thinking skills on students' mathematical reasoning ability, eleventh-grade high school students in a private Chinese high school located in Klang Malaysia, Tee, Leong and Rahim ([15]) found that students with a higher level of critical thinking skills are prone to solve more complex problems. Vice versa, poor performance in mathematics could be attributed to the students' inadequate ability to think critically and analyze mathematical concepts ([16]). To improve students' performance and reasoning ability in mathematics learning, critical thinking needs to be integrated and emphasized in the curriculum ([17],[5]). A long time

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ago, Harold Fawcett, in 1938, introduced the idea that students could learn mathematics through experiences of critical thinking. However, there has been little agreement regarding how critical thinking should be effectively taught.

There are two main approaches to developing general skills in critical thinking, characterized by the development of specialized courses to instruct critical thinking skills. Moore ([18]) intended to accept that students should be taught about critical thinking 'as a subject of study in itself' - critical thinking skills are taught in indirect ways, or through disciplinary studies, but not both. Many types of research support the idea that critical thinking should be regarded as both a general skill and a variety of ways of thinking unique to the subject. Van Gelder, Bisset and Cummings ([19]) are the defenders of the latter disposition, they pointed out that by teaching general reasoning skills, critical thinking skills can be improved. The infusion approach is characterized by the development of these skills by embedding them into the teaching of established learning materials. The "infusion approach" named by Ennis ([20]) integrates critical thinking instruction into the contexts of specific subject matters ([21], [22]). Mathematics is generally considered to be based on rational thinking, and clear and concise language, focusing on the discipline models used to derive conclusions and decision techniques ([23]). In this context, teachers can promote critical thinking skills through discussion and proposing questions. Socratic questioning is one of the strategies used to enhance critical thinking. This type of question is intended to clarify information, identify opinions, discover hypotheses, distinguish between factual claims from value judgments, and identify defects in reasoning ([24]). So, proposing the right question is a learning behaviour that can be taught.

Maričić, Špijunović and Malinović Jovanović ([25]) further discussed the infusion approach in teaching critical thinking in mathematics. According to Maričić et al., ([25]), the method of developing critical thinking focuses on the rules, procedures, and skills of logical thinking that are not recommended for primary students. Maričić et al., ([25]) thought students at this age cannot acquire knowledge through deduction, nor can they use strict logic in thinking, assessment, and learning. Therefore, developing critical thinking must consider the age and ability of the student, and most importantly, it must be based on specific mathematical content ([26]). In other words, we can only develop critical thinking by exercising and putting students in a critical state of thinking. If there is no proper content, strategy, method, form or composition, teaching will not have a major impact on the development of critical thinking in mathematics education ([25]). The infusion approach addresses integrating teaching for thinking into subject areas so that students can understand skills, practice, and apply them in subject contexts. In other words, if the teaching of thinking is clear in the subject area, it will have a great impact on the performance of the students.

Davies ([27]) disagrees with this characterization and believes that critical thinking skills are to some extent general skills, and he recommends combining both approaches to promote critical thinking in teaching. Teachers have a role to facilitate and support students to develop critical thinking skills in mathematics learning. Pedagogical practices and a supportive and collaborative environment are described as effective in fostering critical thinking skills ([28], [29], [30])

2. Models of Teaching and Learning critical thinking

Several models exist that capture the complex process of teaching and learning critical thinking. Bloom's Taxonomy provides a framework for critical thinking. These taxonomies provide a method for preparing instructional units and assessing progress by providing a step-by-step learning process ([31]). According to Benjamin Bloom's Taxonomy of Educational Objectives (1956), an interdisciplinary model for developing higher-order thinking in learning was designed, and six cognitive skills were included- knowledge, comprehension, application, analysis, synthesis, and evaluation. The six-level skills are arranged from lower-level to higher-level learning domains, students need to master each level to demonstrate their development of critical thinking skills (shown in Figure 1). The Cornell/Illinois model is defined and improved by Robert Ennis based on the following definition: Critical thinking is reasonable and reflective thinking focused on deciding what to believe or do ([32]). The model covers three modes of critical thinking skills: induction, deduction, and value judgment. It uses inferential observations, statements, and hypotheses to understand the levels of these three critical thinking skills.

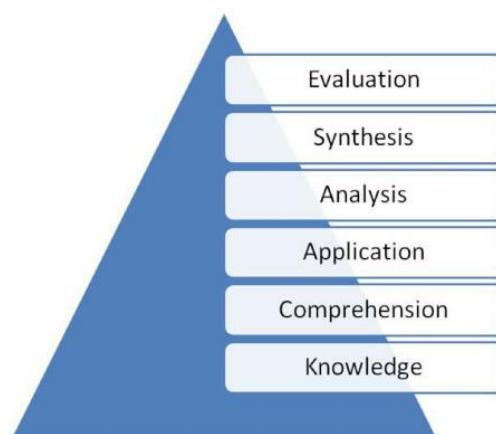


Figure 1: Bloom's Taxonomy ([33], p11)

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The Paul-Elder model was originally developed by Paul ([34],[35]), and further improved by Paul and Elder ([36]). The Paul-Elder model has introduced critical thinking as a part of three key components: reasoning, intellectual standards, and intellectual traits. In the Paul-Elder model, intellectual standards are applied to elements of reasoning and develop students' intellectual characteristics. Students should be able to identify and evaluate their use of elemental reasoning skills. Duron, Limbach and Waugh ([37], p161, shown in Figure Two) designed a five-step model to move students toward critical thinking. Questions at each level may ask students to assess, criticize, recommend, predict, and evaluate. The five-step model requires a commitment to active student-centred learning to motivate students. Creative planning and small class sizes will work productively while applying this model. The above models see learning as a change in the learner's experience or knowledge that results from a change in the learner's environment. Some approaches consider the teaching process only from a pedagogical perspective, while others also consider student input ([37], [31],[38]).

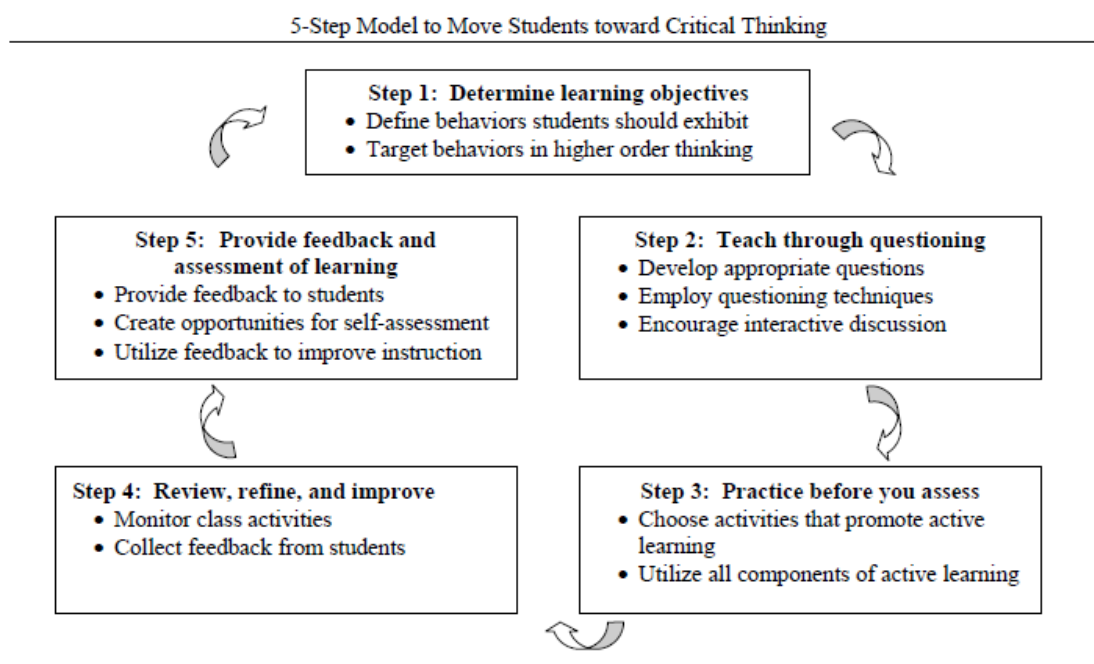


Figure 2: 5-step Model to move students toward critical thinking ([37], p161)

3. Teaching algebra

Algebra is an important branch of secondary mathematics, it can be used in daily life problems, which makes algebra important and necessary content for mathematics education (National Council of Teachers of Mathematics ([39])). Besides that, algebra promotes recognizing and analysing patterns, reasoning, problem-solving skills, and generalizing arithmetic operations through representation with symbols. All these aspects require critical thinking competence. To develop critical thinking, skills specifically logic, patterns, problem-solving, deductive and inductive reasoning, understanding the core concepts of algebra play a critical role. Algebra can help individuals better handle complex problems involving numbers, especially when students enter the workplace in real life. For effective algebra instruction, educators need to recognize the difference between algebra and algebraic thinking in practice. Algebra is the content and product while algebraic thinking is the process. The content includes concepts such as variables, like terms, equivalent algebraic expressions, and equations etc. The contents are the core knowledge that junior secondary students are expected to master before they start advanced algebra. Algebraic thinking consists of understanding series, these series need to explain the problems by converting information or events into mathematical languages, which can then be translated into an algebraic equation. Through generalizing patterns, presenting relationships, and analyzing visible changes, algebraic thinking can link various mathematical topics to gain a better understanding ([40]), students need the ability to understand series to solve problems with algebra knowledge. Several studies reveal that students' abilities are still insufficient when dealing with generational activities in algebra ([41],[33],[42]). Research found that most students do not understand the concept of algebra and they also think that learning algebra is not fun at all ([43]). Some Junior students can generalize patterns and use symbols ([44],[45]), while most students face challenges when they answer questions that require understanding and application of algebra to perform complex procedures ([46]).

There are different methods or models for teaching algebra efficiently. For example, Augmented Reality (AR) was applied to junior high school students to promote their algebra learning ([43]). The research found that the students with high self-efficacy attained higher scores than those with low self-efficacy as they may have more engagement and involvement in the learning activity. Since there are no sufficient facilities, it is not possible to apply in rural areas to learn based on AR learning ([43]). Siew

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et al., ([46]) explored applying a video game app to impact students' algebraic learning, and the research result showed that the app can motivate students positively in their learning and make them more confident in learning algebra. To obtain satisfactory learning outcomes, teachers need to modify to make sure the game design is related to the content and students are more focused on learning mathematics than just playing games. In the model of teaching algebra, teachers spent time on whole-group instructions or group work, and teaching tended to follow the textbook ([47],[48]).

Previous research discussed students' algebraic thinking, focusing on algebraic thinking and learning, processes of using arithmetical thinking when solving problems and motivating students in algebraic learning ([49],[50],[51],[52]). Students' abilities in each component of algebraic thinking need critical thinking competence. However, how to develop students' critical thinking skills in algebra learning lacks exploration, the gap is there are few studies about models that teach or support students to develop this. Thus, it would be worthwhile to design a model that can establish a link between algebraic learning and the promotion of critical thinking.

III. RESEARCH QUESTIONS

The goal of this study is to address the lack of information on the construction of the connection between critical thinking skills and algebra learning. By doing so, this study explored teachers' perspectives on learning algebra and what critical thinking skills students should master, then found the connection between them. A model was further designed to support students in developing critical thinking skills in their algebra learning. The model would provide guidelines for pedagogical practice in secondary mathematics algebra. Research questions explored in this study are:

1. What are teachers' perceptions of relative critical thinking with algebra learning?
2. What are the challenges in learning algebra?
3. How to design an innovative model to promote critical thinking in algebra learning?

IV. METHODOLOGY

Research design

This research aimed to design a learning model with the implementation of algebraic learning to increase critical thinking skills for Year 7 students. Quantitative and qualitative mixed methods were applied in this phase but with different weighting. The research followed the phases of the development (shown in Table 1): the initial investigation phases; exploring the critical thinking skills in mathematics education ([5]), and the Australian Mathematics Curriculum and Victorian Mathematics Curriculum, a conception diagram that connects critical thinking skills and algebra learning was provided (shown in Figure 1). Subsequently, a questionnaire was written, which included all the skills in the connection diagram. Data collection was done by a semi-interview to provide an accurate description of a real situation. Ten secondary mathematics teachers participated in the questionnaire and semi-interview to collect data on what kind of critical thinking skills students used when solving algebraic questions, and the challenges that students face when learning algebra. Then, by comparing the two data, the skills that the model needs to cover were identified. A model was further designed based on this information.

Table 1: Research Design

The initial investigation phases	A conception diagram between critical thinking and algebra learning.
A questionnaire A semi-interview	Collect data about the connection between critical thinking skills and algebraic skills, the challenges in learning algebra
Design a learning model	Explore an effective method for supporting students to develop critical thinking skills via learning algebra
Interview	Interview teachers to get feedback on the model

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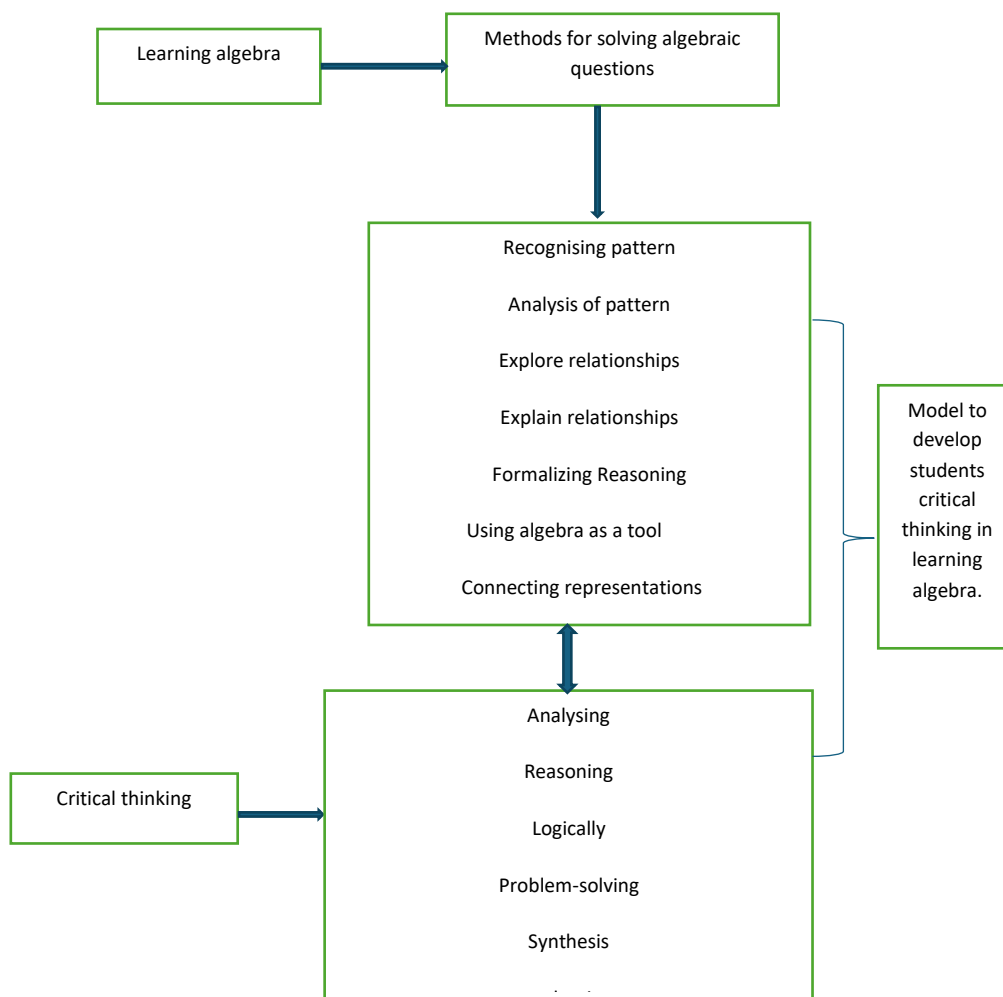


Figure 1. Conceptual Diagram

Participants

This study was conducted in a Christian college located in Victoria Australia. Ten secondary mathematics teachers shared their experiences of critical thinking and algebraic learning via a questionnaire, semi-interview and interview.

V. RESULTS AND DISCUSSION

Question 1: What are teachers' perceptions of relative critical thinking with algebra learning?

Educators have their preferred ways of preparing what they plan/design, observe, remember, and think about when developing critical thinking. It can be identified as students develop and enhance their critical thinking skills if they can solve problems through alternatives and simplified processes. The initial investigation phase aimed to explore information, and then distinguish relevant and irrelevant skills between critical thinking and algebra learning.

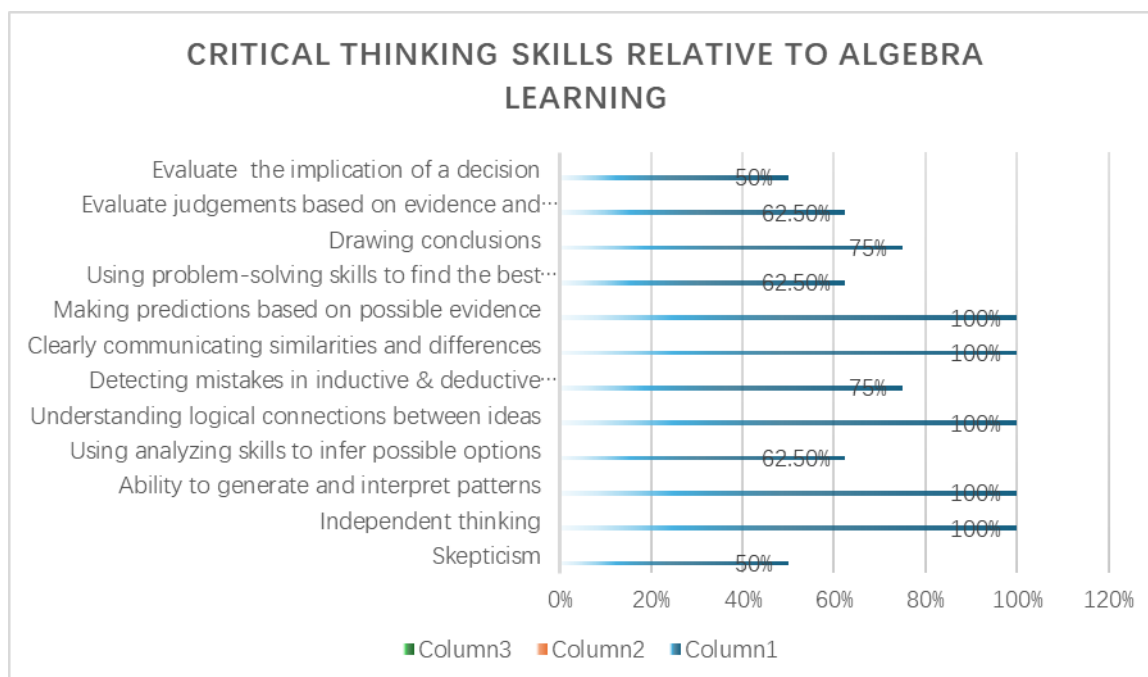


Figure 2 Critical thinking skills relative to algebra learning

Regarding skills related to algebra learning, the data showed that all participants valued the following five skills: independent skills, the ability to generate and interpret patterns, understanding logical connections between ideas, clearly communicating similarities and differences, and using problem-solving skills to find the best solutions. When students do algebra questions, they need to provide a workout to demonstrate their understanding, this process not only shows written results but also gives some insight into students' mathematical thinking. Independent thinking is the process of learning mathematics, and it is necessary to provide students with a productive space for struggle ([53]). The struggle can be by any algebra questions, which are student-centred with teachers providing support for students to discuss and solve. This pedagogical practice will take time to see the achievement, but students can explore their thinking, and compare their results with others, which facilitates their independent thinking in learning. Creating algebra patterns needs students' interpreting and reasoning skills, to obtain these skills, students apply instructions by observing whether the patterns are visual like shapes, colours or letters; and by calculating and reasoning whether the patterns are numbers or variables. Repeating examples many times till they understand how to generate the patterns. When confronted with challenging questions, students need to go through the steps or methods they have learned, understand and identify logical connections between the consecutive terms in a pattern, to develop reasoning skills like induction or deduction. When students can build a connection between two terms, they can form the same or similar patterns independently. These skills will be carried out when they construct familiar mathematical properties in algebra learning ([54]). Analysing similarities and differences requires students to explain their results confidently, reasonably and correctly, students are expected to be trained in solving problems through those reasoning processes ([52]), and communication skills are vital in developing critical thinking skills. Students are encouraged to have an opportunity "to explain their ideas and share their findings, allowing students to reflect on their interpretations, recognize conflicts and contradictions, and consider the mathematical qualities of specific strategies and models" ([55], p7152), teachers should provide the mathematical communication environment to facilitate students to develop their communication skills in the classroom. Participants in the questionnaire signify all the criteria skills that learning algebra is not only a mathematical task but also a mathematical activity, as a task means educators focus on students' answers while the activity is more about the process.

2. What are the challenges in learning algebra?

The quantitative data obtained from the questionnaire was used to collect perceptions of the challenges in learning algebra, from the contents and pedagogical practice aspects. Coding took the form of identifying the rank of challenges by one digit number, for example, the least challenge is coded as 1, and in increasing order, the most challenge one is coded as 6.

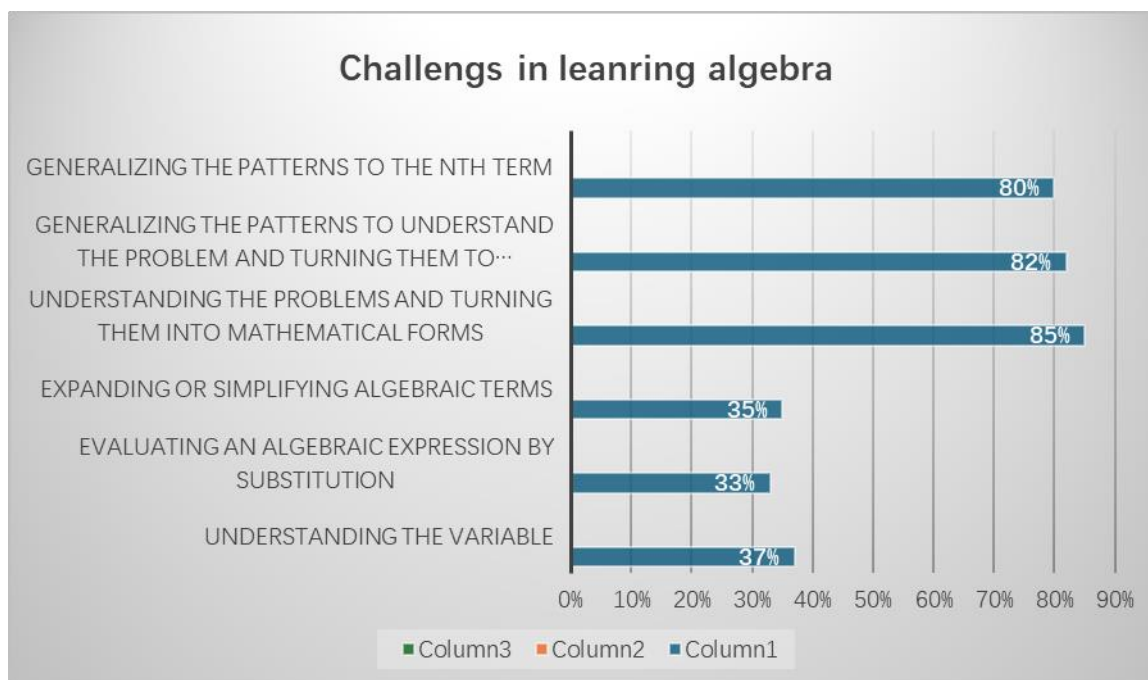


Figure 3 challenges in learning algebra

Figure 3 shows the rank of challenges in learning algebra. The top three were understanding the problems and turning them into mathematical forms (85%), generalizing the patterns to understand the problem and turning them into mathematical forms (82%) and generalizing the patterns to the nth term (80%).

Table 2 Means and Standard Deviations (SD) for challenges in algebra learning

Challenges in algebra learning	N	valid	mean	SD
Understanding the variable	10	22	2.2	1.4
Evaluating an algebraic expression by substitution	10	20	2.2	0.632
Expanding or simplifying algebraic terms	10	21	2.1	0.831
Understanding the problems and turning them into mathematical forms	10	51	5.1	0.943
Generalizing the patterns to understand the problem and turning them into mathematical forms	10	49	4.9	0.831
Generalizing the patterns to the nth term	10	48	4.8	0.872

As can be seen from Table 2, participants expressed their strong agreement (M=5.1) with the challenge of ‘understanding the problems and turning them into mathematical forms’, with the second highest mean (M=4.9) being for ‘generalizing the patterns to understand the problem and turning them into mathematical forms’. With a high mean score of (M=4.8), participants agreed that ‘generalizing the patterns to the nth term’ is the third highest challenge. Translating problems into mathematical form requires students to shift their thinking from arithmetic to a more abstract and logical way of processing information. This process is not just about memorizing formulas or applying them, but about decoding the problem and identifying relevant information, further understanding the complex relationships between variables and other values, about how to figure out the relationships between terms, and formulating an equation or set of equations. To complete this process requires critical thinking skills such as analysing, interpreting, logical thinking and summarizing skills.

3. How to design an innovative model to promote critical thinking in algebra learning?

The evaluation of mathematical critical thinking skills must focus on students' thinking processes when solving mathematical problems, not just the results of the answers. The model aims to further support students in developing critical thinking skills in

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algebra learning. It is designed based on the collected data from research questions 1 and 2, focused on critical thinking skills that students need for solving algebra questions after considering the level of difficulties for Year 7 students (shown in Table 3). A sample of critical thinking of learning objectives (shown in Table 4) was provided as an example when considering how to design the model (shown in Figure 4).

Table 3. Methods for solving algebra questions and critical thinking skills

Methods for solving algebra questions	Connected to critical thinking skills
Recognition of pattern	analysing
Analysing of pattern	Analysing, reasoning and logically
Reasoning	Logically and reasoning
problem-solving	Problem-solving synthesis
Generalizing arithmetic operation	Representing, clarifying ideas and evaluation

Table 4. A sample of critical thinking learning objectives for algebraic questions

Critical thinking skill	Objectives
comprehension	Identify two basic approaches to solving algebraic equations
analysis	Compare different methods in simplifying algebraic expressions and give possible reasons for that
reasoning	Provide a logical explanation of the solution of an algebraic word problem
synthesis	Based on the algebraic analysis, identify actions that might be taken to improve the arithmetic operation. Detect any mistakes or self-mark your errors.
evaluation	While solving algebraic questions, assess the overall methods, correction of solution and problem-solving ability. Your recommendation to extend the utilization of problem-solving strategies in an application.

Critical thinking is a way of thinking that tests relations, and evaluates aspects of a problem, including the ability to gather information and analyze situations, to understand and identify what is necessary, therefore critical thinking is analytical thinking. Critical thinking is not an innate ability that a student can develop at birth. According to Liu ([5]), students are not born with the ability to think critically, and previous learning experiences do not require them to think critically. Therefore, in the classroom, teachers must understand student's learning behaviour and apply the model gradually. This model is designed by development procedures that pay attention to the characteristics and needs of students. Data from this study provides different aspects of how to learn algebra critically and effectively. Students who are struggling with some basic skills, really need to practice questions with those basic skills before teachers address critical thinking skills. Students will become critical and creative users of mathematics when they develop four effective mathematical processes: comprehension, fluency, problem-solving, and reasoning (Australian Curriculum, Assessment and Reporting Authority ([6]). To understand mathematics correctly, students need to develop critical thinking skills ([56]) as they are expected to be logical, critical, analytical, creative, careful and thorough and responsive in solving problems.

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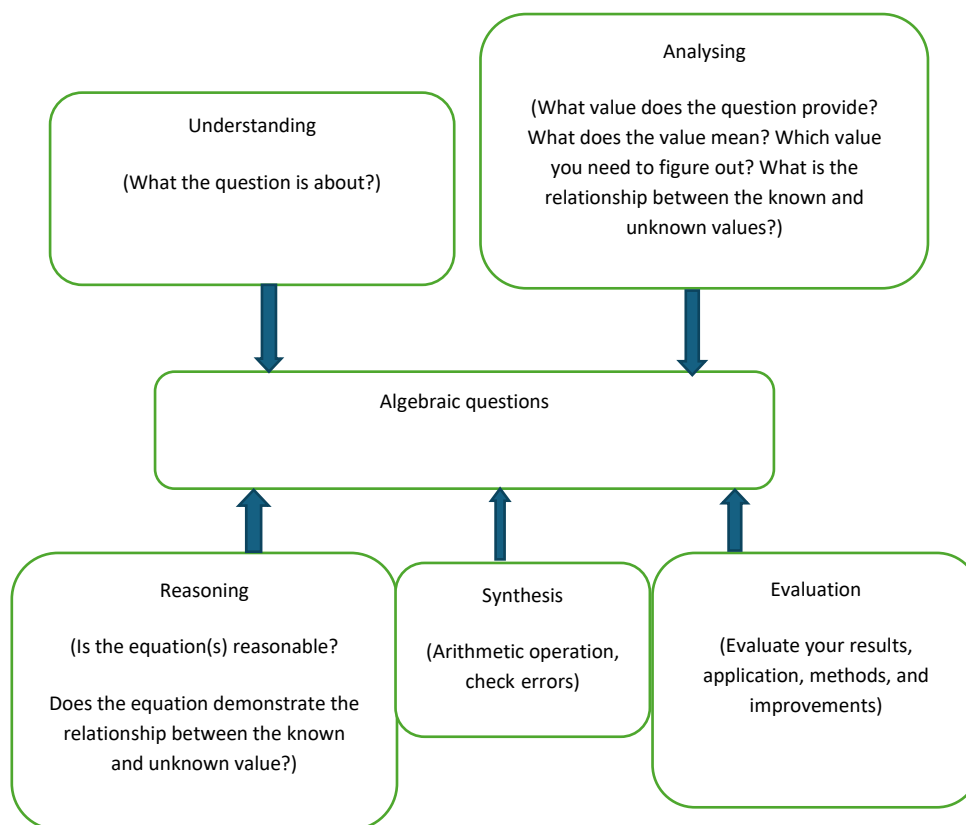


Figure 4. Critical- algebraic Model

VI. CONCLUSION

Findings

The data from this study yielded several findings related to critical thinking skills in algebra learning. First, relative critical thinking skills to algebra learning were discussed. Independent skills, the ability to generate and interpret patterns, understanding logical connections between ideas, clearly communicating similarities and differences, and using problem-solving skills to find the best solutions were identified to be important skills for students to master algebra. Second, challenges in learning algebra were stated. The awareness of the challenges of learning algebra makes teachers focus on the weaknesses in their planning and teaching, also, this will ensure students get enough practice with the new knowledge. Third, pedagogical knowledge was analyzed to tend to provide direction in teaching and learning. Creating a critical-thinking classroom environment will provide students with more opportunities to be involved in solving problems with thinking skills, understanding students' thinking behaviour and learning style will help teachers' future planning to achieve effective teaching and learning outcomes. Fourth, this study contributed an innovative model designed to develop critical thinking skills in algebra learning, which has not yet been studied intensively by previous researchers. It is hoped that teachers will take the initiative to encourage students' creative critical thinking behaviour in learning algebra. It is expected that teachers will be able to utilize the model to develop their teaching methods in algebra.

Limitations and future research

When exploring the connection between critical thinking and algebra learning, this study is only focused on generational algebra skills. To further understand students' difficulties, the circumstances and causes of difficulties need to be analyzed. Future research is needed to investigate, an effective way to simultaneously achieve a balance of mathematical thinking and effective learning, as well as the validity of the model. Because of the small sample size of teacher participants in this study, other studies with broader samples must be considered to interpret the results of this study.

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